

Influence of the beam-size or MD-effect on particle losses at B-factories PEP-II and KEKB

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For the $e^+e^- \rightarrow e^+e^-\gamma$ process at colliding beams, macroscopically large impact parameters give an essential contribution to the standard cross section. These impact parameters may be much larger than the transverse sizes of the colliding bunches. It means that the standard calculations have to be essentially modify. In the present paper such a beam-size or MD-effect is calculated for bremsstrahlung at B-factories PEP-II and KEKB using the list of nominal parameters from Review of Particle Physics (2002). We find out that this effect reduces beam losses due to bremsstrahlung by about 20%.

Key words: B-factories, beam-size effect, beam losses

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1 Introduction: beam-size or MD-effect

The so called beam-size or MD-effect is a phenomenon discovered in experiments [1] at the MD-1 detector (the VEPP-4 accelerator with e^+e^- colliding beams, Novosibirsk 1981). It was found out that for ordinary bremsstrahlung, macroscopically large impact parameters should be taken into consideration. These impact parameters may be much larger than the transverse sizes of the interacting particle bunches. In that case, the standard calculations, which do not take into account this fact, will give incorrect results. The detailed description of the MD-effect can be found in review [2].

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We start with a few words about history of this effect. In 1980–1981 a dedicated study of the process $e^+e^- \rightarrow e^+e^-\gamma$ has been performed at the collider VEPP-4 in Novosibirsk using the detector MD-1 for an energy of the electron and positron beams $E_e = E_p = 1.8$ GeV and in a wide interval of the photon energy E_γ from 0.5 MeV to $E_\gamma \approx E_e$. It was observed [1] that the number of measured photons was smaller than that expected. The deviation from the standard calculation reached 30% in the region of small photon energies and vanished for large energies of the photons. A. Tikhonov [3] pointed out that those impact parameters ϱ , which give an essential contribution to the standard cross section, reach values of $\varrho_m \sim 5$ cm whereas the transverse size of the bunch is $\sigma_\perp \sim 10^{-3}$ cm. The limitation of the impact parameters to values $\varrho \lesssim \sigma_\perp$ is just the reason for the decreasing number of observed photons.

The first calculations of this effect have been performed in Refs. [4] and [5] using different versions of quasi-classical calculations in the region of large impact parameters. Further experiments, including the measurement of the radiation probability as function of the beam parameters, supported the concept that the effect arises from the limitation of the impact parameters. Later on, the effect of limited impact parameters was taken into account when the single bremsstrahlung was used for measuring the luminosity at the VEPP-4 collider [6] and at the LEP-I collider [7]. In the case of the VEPP-4 experiment [6], it was checked that the luminosities obtained using either this process or using other reactions (such as the double bremsstrahlung process $e^+e^- \rightarrow e^+e^-\gamma\gamma$, where the MD-effect is absent) agreed with each other.

A general scheme to calculate the finite beam size effect had been developed in paper [8] starting from the quantum description of collisions as an interaction of wave packets forming bunches. Since the effect under discussion is dominated by small momentum transfer, the general formulae can be considerably simplified. The corresponding approximate formulae were given. They are obtained from an analysis of Feynman diagrams and it allows to estimate the accuracy of approximation. In a second step, the transverse motion of the particles in the beams can be neglected. The less exact (but simpler) formulae, which are then found, correspond to the results of Refs. [4] and [5]. It has also been shown that similar effects have to be expected for several other reactions such as bremsstrahlung for colliding ep -beams [9], [10], e^+e^- -pair production in $e^\pm e$ and γe collisions [8]. The corresponding corrections to the standard formulae are now included in programs for simulation of events at linear colliders. The influence of MD-effect on polarization had been considered in Ref. [11].

In 1995 the MD-effect was experimentally observed at the electron-proton collider HERA [12] on the level predicted in [10].

The possibility to create high-energy colliding $\mu^+\mu^-$ beams is now wildly dis-

cussed. For several processes at such colliders a new type of beam-size effect will take place — the so called linear beam-size effect [13]. The calculation of this effect had been performed by method developed for MD-effect in [8].

It was realized in last years that MD-effect in bremsstrahlung plays important role for the problem of beam lifetime. At storage rings TRISTAN and LEP-I, the process of a single bremsstrahlung was the dominant mechanism for the particle losses in beams. If electron loses more than 1 % of its energy, it leaves the beam. Since MD-effect reduced considerable the effective cross section of this process, the calculated beam lifetime in these storage rings was larger by about 25 % for TRISTAN [14] and by about 40 % for LEP-I [15] (in accordance with the experimental data) then without taken into account the MD-effect.

In next Section we present the qualitative description of the MD-effect. In Sect. 3 we calculate the MD-effect and its influence on the beam losses at the existing B-factories. We find out that this effect reduces beam losses due to bremsstrahlung by about 20%.

At the end of this section we also mention about recent paper [16] in which previous results [4], [5], [8] about bremsstrahlung spectrum had been revised. It was claimed that an additional subtraction related to the coherent contribution has to be done. However, this additional subtraction is negligible small for the real parameters of the existing B-factories. Besides, paper [16], in our opinion, is incorrect. In our critical remark [17] we analyzed in detail the coherent and incoherent contributions in the conditions, considered in paper [16], and, in contrast to above claims, we found out that under these conditions the coherent contribution is completely negligible and, therefore, there is no need to revise the previous results.

2 Qualitative description of the MD-effect

Qualitatively we describe the MD-effect using as an example the $ep \rightarrow ep\gamma$ process³. This reaction is defined by the diagrams of Fig. 1 which describe the radiation of the photon by the electron (the contribution of the photon radiation by the proton can be neglected). The large impact parameters $\varrho \gtrsim \sigma_\perp$, where σ_\perp is the transverse beam size, correspond to small momentum transfer $\hbar q_\perp \sim (\hbar/\varrho) \lesssim (\hbar/\sigma_\perp)$. In this region, the given reaction can be

³ Below we use the following notations: N_e and N_p are the numbers of electrons and protons (positrons) in the bunches, σ_H and σ_V are the horizontal and vertical transverse sizes of the proton (positron) bunch, $\gamma_e = E_e/(m_e c^2)$, $\gamma_p = E_p/(m_p c^2)$ and $r_e = e^2/(m_e c^2)$ is the classical electron radius.

represented as a Compton scattering (Fig. 2) of the equivalent photon, radiated by the proton, on the electron. The equivalent photons with frequency ω form a “disk” of radius $\varrho_m \sim \gamma_p c / \omega$ where $\gamma_p = E_p / (m_p c^2)$ is the Lorentz-factor of the proton. Indeed, the electromagnetic field of the proton is γ_p -times contracted in the direction of motion. Therefore, at distance ϱ from the axis of motion a characteristic longitudinal length of a region occupied by the field can be estimated as $\lambda \sim \varrho / \gamma_p$ which leads to the frequency $\omega \sim c / \lambda \sim \gamma_p c / \varrho$.

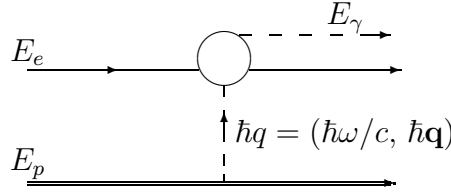


Fig. 1. Block diagram of radiation by the electron.

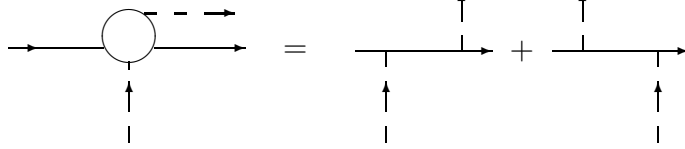


Fig. 2. Compton scattering of equivalent photon on the electron.

In the reference frame connected with the collider, the equivalent photon with energy $\hbar\omega$ and the electron with energy $E_e \gg \hbar\omega$ move toward each other (Fig. 3) and perform a Compton scattering. The characteristics of this process are well known. The main contribution to the Compton scattering is given by the region where the scattered photons fly in a direction opposite to that of the initial photons. For such a backward scattering, the energy of the equivalent photon $\hbar\omega$ and the energy of the final photon E_γ and its emission angle θ_γ are related by

$$\hbar\omega = \frac{E_\gamma}{4\gamma_e^2(1 - E_\gamma/E_e)} [1 + (\gamma_e\theta_\gamma)^2] \quad (1)$$

and, therefore,

$$\hbar\omega \sim \frac{E_\gamma}{4\gamma_e^2(1 - E_\gamma/E_e)}. \quad (2)$$

As a result, we find the radius of the “disk” of equivalent photons with the frequency ω (corresponding to a final photon with energy E_γ) as follows:

$$\varrho_m = \frac{\gamma_p c}{\omega} \sim \lambda_e 4\gamma_e \gamma_p \frac{E_e - E_\gamma}{E_\gamma}, \quad \lambda_e = \frac{\hbar}{m_e c} = 3.86 \cdot 10^{-11} \text{ cm}. \quad (3)$$

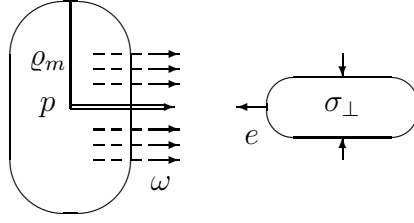


Fig. 3. Scattering of equivalent photons, forming the “disk” with radius ϱ_m , on the electron beam with radius σ_\perp .

For the HERA collider with $E_p = 820$ GeV and $E_e = 28$ GeV one obtains

$$\varrho_m \gtrsim 1 \text{ cm} \quad \text{for} \quad E_\gamma \lesssim 0.2 \text{ GeV} . \quad (4)$$

Equation (3) is also valid for the $e^-e^+ \rightarrow e^-e^+\gamma$ process with replacement protons by positrons. For the VEPP-4 collider it leads to

$$\varrho_m \gtrsim 1 \text{ cm} \quad \text{for} \quad E_\gamma \lesssim 15 \text{ MeV} , \quad (5)$$

for the PEP-II and KEKB colliders we have

$$\varrho_m \gtrsim 1 \text{ cm} \quad \text{for} \quad E_\gamma \lesssim 0.1 \text{ GeV} . \quad (6)$$

The standard calculation corresponds to the interaction of the photons forming the “disk” with the unbounded flux of electrons. However, the particle beams at the HERA collider have finite transverse beam sizes of the order of $\sigma_\perp \sim 10^{-2}$ cm. Therefore, the equivalent photons from the region $\sigma_\perp \lesssim \varrho \lesssim \varrho_m$ cannot interact with the electrons from the other beam. This leads to the decreasing number of the observed photons and the “observed cross section” $d\sigma_{\text{obs}}$ is smaller than the standard cross section $d\sigma$ calculated for an infinite transverse extension of the electron beam,

$$d\sigma_{\text{obs}} = d\sigma - d\sigma_{\text{cor}} . \quad (7)$$

Here the correction $d\sigma_{\text{cor}}$ can be presented in the form

$$d\sigma_{\text{cor}} = d\sigma_C(\omega, E_\gamma) dn(\omega) \quad (8)$$

where $dn(\omega)$ denotes the number of “missing” equivalent photons and $d\sigma_C$ is the cross section of the Compton scattering. Let us stress that the equivalent photon approximation in this region has a high accuracy (the neglected terms are of the order of $1/\gamma_p$). But for the qualitative description it is sufficient to

use the logarithmic approximation in which this number is (see[18], §99)

$$dn = \frac{\alpha}{\pi} \frac{d\omega}{\omega} \frac{dq_{\perp}^2}{q_{\perp}^2}. \quad (9)$$

Since $q_{\perp} \sim 1/\varrho$, we can present the number of “missing” equivalent photons in the form

$$dn = \frac{\alpha}{\pi} \frac{d\omega}{\omega} \frac{d\varrho^2}{\varrho^2} \quad (10)$$

with the integration region in ϱ :

$$\sigma_{\perp} \lesssim \varrho \lesssim \varrho_m = \frac{\gamma_p c}{\omega}. \quad (11)$$

As a result, this number is equal to

$$dn(\omega) = 2 \frac{\alpha}{\pi} \frac{d\omega}{\omega} \ln \frac{\varrho_m}{\sigma_{\perp}}, \quad (12)$$

and the correction to the standard cross section with logarithmic accuracy is (more exact expression is given by Eq. (18)) ⁴

$$d\sigma_{\text{cor}} = \frac{16}{3} \alpha r_e^2 \frac{dy}{y} \left(1 - y + \frac{3}{4}y^2\right) \ln \frac{4\gamma_e \gamma_p (1-y)\lambda_e}{y\sigma_{\perp}}, \quad y = \frac{E_{\gamma}}{E_e}. \quad (13)$$

3 MD-effect for PEP-II and KEKB

Usually in experiments the cross section is found as the ratio of the number of observed events per second $d\dot{N}$ to the luminosity L . Also, in our case it is convenient to introduce the “observed cross section”, defined as the ratio

$$d\sigma_{\text{obs}} = \frac{d\dot{N}}{L}. \quad (14)$$

⁴ Within this approximation, the standard cross section has the form (more exact expression is given by Eq. (17))

$$d\sigma = d\sigma_C \frac{\alpha}{\pi} \frac{d\omega}{\omega} \frac{dq_{\perp}^2}{q_{\perp}^2} = \frac{16}{3} \alpha r_e^2 \frac{dy}{y} \left(1 - y + \frac{3}{4}y^2\right) \ln \frac{4\gamma_e \gamma_p (1-y)}{y}$$

with the integration region $\hbar\omega/(c\gamma_p) \lesssim \hbar q_{\perp} \lesssim m_e c$ corresponding to the impact parameters ϱ in the interval $\lambda_e \lesssim \varrho \lesssim \varrho_m$.

Contrary to the standard cross section $d\sigma$, the observed cross section $d\sigma_{\text{obs}}$ depends on the parameters of the beams which scatter. To indicate explicitly this dependence we introduce the “correction cross section” $d\sigma_{\text{cor}}$ as the difference between $d\sigma$ and $d\sigma_{\text{obs}}$:

$$d\sigma_{\text{obs}} = d\sigma - d\sigma_{\text{cor}}. \quad (15)$$

The relative magnitude of the MD-effect is given, therefore, by quantity

$$\delta = \frac{d\sigma_{\text{cor}}}{d\sigma}. \quad (16)$$

Let us consider the number of photons emitted by electrons in the process $e^-e^+ \rightarrow e^-e^+\gamma$. The standard cross section for this process is well known:

$$d\sigma = \frac{16}{3}\alpha r_e^2 \frac{dy}{y} \left(1 - y + \frac{3}{4}y^2\right) \left[\ln \frac{4\gamma_e\gamma_p(1-y)}{y} - \frac{1}{2} \right], \quad y = \frac{E_\gamma}{E_e}. \quad (17)$$

The correction cross section is given by expression

$$d\sigma_{\text{cor}} = \frac{16}{3}\alpha r_e^2 \frac{dy}{y} \left[\left(1 - y + \frac{3}{4}y^2\right) L_{\text{cor}} - \frac{1-y}{12} \right] \quad (18)$$

where

$$L_{\text{cor}} = \ln \frac{2\sqrt{2}\gamma_e\gamma_p(1-y)(a_H + a_V)\lambda_e}{a_H a_V y} - \frac{3+C}{2},$$

$$\lambda_e = \frac{\hbar}{m_e c} = 3.86 \cdot 10^{-11} \text{ cm}, \quad (19)$$

$C = 0.577\dots$, quantities $a_H = \sqrt{\sigma_{eH}^2 + \sigma_{pH}^2}$ and $a_V = \sqrt{\sigma_{eV}^2 + \sigma_{pV}^2}$ related to the r.m.s. transverse horizontal and vertical bunch sizes σ_{jH} and σ_{jV} for the electron, $j = e$, and positron, $j = p$, beams. In calculations we used data from Review of Particle Physics–2002 [19] (see Table 1).

Table 1

	E_e , GeV	E_p , GeV	σ_V , μm	σ_H , μm	Energy spread, %	L , 10^{33} $\text{cm}^{-2} \text{s}^{-1}$	N_e , 10^{10}	n_b	τ_L , hr
PEP-II	9	3.1	4.7	157	0.061	4.6	2.1	800	2.5
KEKB	8	3.5	2.7	110	0.07	7.25	4.5	1224	3.4

The observed number of photons is smaller due to MD-effect than the number of photons calculating without this effect. The relative magnitude of MD-effect

is given by quantity δ from Eq. (16) (see Table 2). It is seen that MD-effect reduced considerable the differential cross section.

Table 2

$y = E_\gamma/E_e$	0.001	0.005	0.01	0.05	0.1	0.5
δ , % PEP-II	31	26	24	19	16	6.0
δ , % KEKB	33	29	26	21	18	8.9

To estimate the integrated contribution of the discussed process into particle losses, we should integrate the differential observed cross section from some minimal photon energy. It is usually assumed that an electron leaves the beam when it emits the photon with the energy 10 times larger than the beam energy spread. In other words, the relative photon energy should be $y = E_\gamma/E_e \geq y_{\min}$ where $y_{\min} = 0.0061$ for PEP-II and $y_{\min} = 0.007$ for KEKB. After integration of the observed cross section from $y_{\min} \ll 1$ up to $y_{\max} = 1$, we obtain

$$\sigma_{\text{obs}} = \frac{16}{3} \alpha r_e^2 \left\{ \left(\ln \frac{1}{y_{\min}} - \frac{5}{8} \right) \left[\ln \frac{\sqrt{2} a_H a_V}{(a_H + a_V) \lambda_e} + \frac{2 + C}{2} \right] + \frac{1}{12} \left(\ln \frac{1}{y_{\min}} - 1 \right) \right\}. \quad (20)$$

Let us note that the standard cross section integrated over the same interval of y ,

$$\sigma = \frac{16}{3} \alpha r_e^2 \left\{ \left(\ln \frac{1}{y_{\min}} - \frac{5}{8} \right) \left[\ln (4 \gamma_e \gamma_p) - \frac{1}{2} \right] + \frac{1}{2} \left(\ln \frac{1}{y_{\min}} \right)^2 - \frac{3}{8} - \frac{\pi^2}{6} \right\}, \quad (21)$$

is larger than the observed cross section by about 20 % (see Table 3).

To understand the importance of the bremsstrahlung channel for particle losses, we estimate the corresponding partial beam lifetime. The number of particles, which the single electron bunch losses during a second, equals to

$$\Delta \dot{N}_e = L \sigma_{\text{obs}} / n_b \quad (22)$$

where L is a luminosity and n_b is the number of bunches. Therefore, the partial lifetime of the electron bunch, corresponding to bremsstrahlung process at a given luminosity, can be estimated as

$$\tau_{\text{brem}}^e = \frac{N_e}{\Delta \dot{N}_e} = \frac{N_e n_b}{L \sigma_{\text{obs}}}. \quad (23)$$

The obtained numbers for the electron and positron beams are presented in Table 3. They can be compared with the luminosity lifetime τ_L from Table 1 which is some average characteristics of lifetimes for both beams. More detailed comparison with the experimental numbers for lifetimes of beams at KEKB shows that the bremsstrahlung process is important for the electron beam lifetime, but has rather small influence on the positron beam lifetime.

Table 3

	$\sigma_{\text{obs}}, 10^{-25} \text{ cm}^2$	$\sigma/\sigma_{\text{obs}}$	$\tau_{\text{brem}}^e, \text{ hr}$	$\tau_{\text{brem}}^p, \text{ hr}$
PEP-II	2.5	1.20	4	12
KEKB	2.4	1.23	8.9	14

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